Phase Lock Loop Stability Analysis

This article presents a review of PLL transfer functions with attention to the conditions required for steady-state stability

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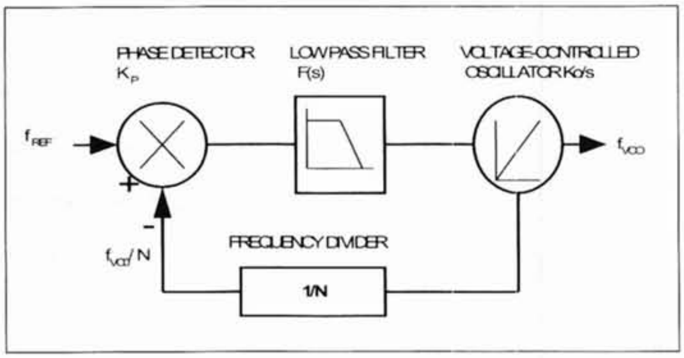
Phase lock loops (PLLs) are key components of modern communication systems. Frequency synthesizers, FM demodulators and clock recovery circuits are some applications of PLLs. An important aspect of PLL design is the steady state stability of the loop. This article examines this aspect of PLL design, particularly the effect of loop filter on PLL stability.

PLLs are negative feedback control systems comprising of a phase-frequency detector (PD), a loop filter, a voltage-controlled oscillator (VCO) and a frequency divider. The function of the PD is to generate an output waveform based on the difference in phase (and frequency) between the input signal and a fixed reference. This is followed by a loop filter, normally a lowpass filter (LPF), whose function is to filter out any high frequency harmonics from the phase detector and to provide a DC signal output; followed by a VCO that generates a high frequency signal controlled by the DC input signal. A sample of the VCO output signal is then fed back to the input of the PD as the input waveform and compared in phase (and frequency) to a fixed reference. In a locked condition, the PLL output signal is locked in phase (and frequency) to the fixed input reference, i.e. the output phase has a fixed differential from the input phase.

According to control loop theory, the transfer function of the PLL is (assuming N = 1):

$$(\theta_1 - \theta_0)K_P F(s)K_0 / s = \theta_0$$
$$(1 + K_P F(s)K_0 / s)\theta_0 = \theta_1 K_P F(s)K_0 / s \quad (1)$$



▲ **Figure 1. Block diagram of a PLL.**

$$T(s) = \frac{\theta_0}{\theta_1} = \frac{K_P F(s) K_0 / s}{\left(1 + K_P F(s) K_0 / s\right)} = \frac{K_P F(s) K_0}{s + K_P F(s) K_0} = \frac{G(s)}{1 + G(s)}$$

where

$T(s)$ is the closed loop PLL transfer function in the frequency domain (θ_I and θ_0 being the input and output signal respectively),

$G(s) = K_P F(s) K_0 / s$ is the open loop transfer function (complex) of the PLL,

K_P is the transfer function of the phase detector in Volts/ Hz,

K_0/s is the transfer function of the VCO in Hz/ volts, and

$F(s)$ is the transfer function of the loop filter.

Note that this is the transfer function of the PLL when the loop is closed. The block diagram of the PLL is shown in Figure 1.

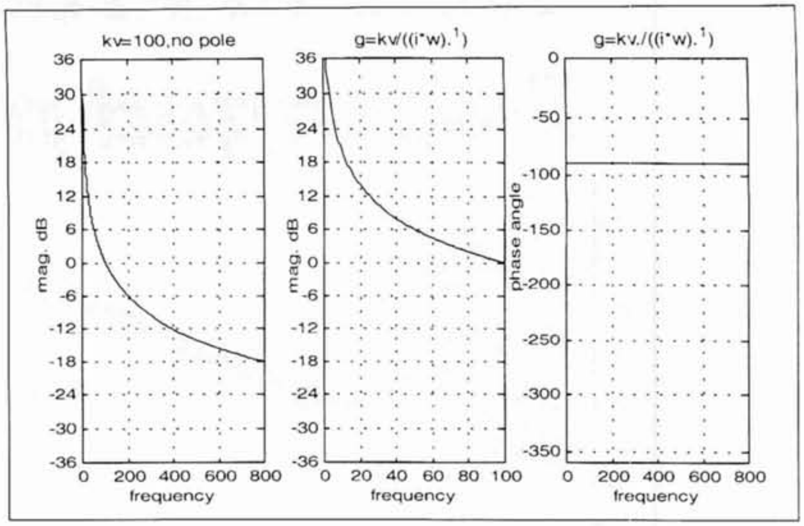
Before we examine the PLL closed loop transfer function in detail, it is important to examine the stability of the PLL. A PLL is unstable when the denominator of the closed loop transfer function is equal to zero. For this to occur,

$$1 + G(s) = 0$$

$$G(s) = -1, \text{ or} \qquad (2)$$

$$G(s) = 0 \text{ dB} @ \angle -180° \text{ (magnitude/phase angle)}$$

Hence, the PLL is unstable at the frequency where the magnitude of the open loop transfer function is unity and the phase angle is –180 degrees. Because stability is an important aspect of any PLL design, the condition of unity open loop gain and a phase angle of 180 degrees must be avoided.

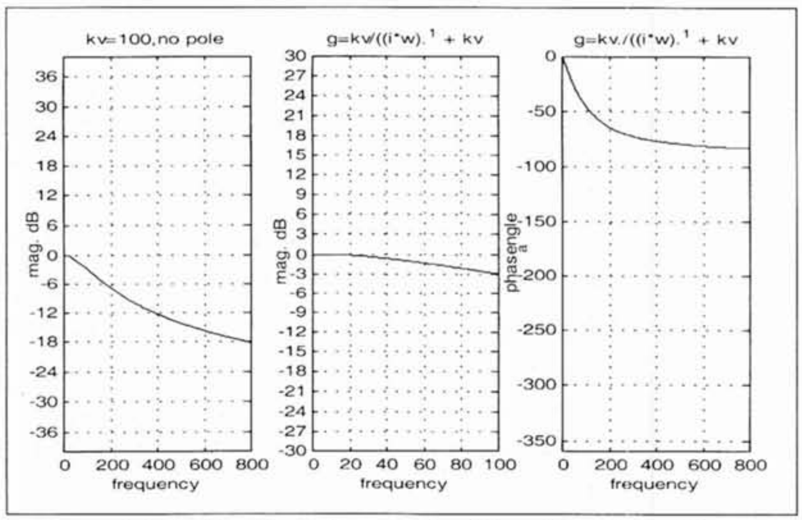


▲ **Figure 2. PLL open loop transfer function with no loop filter.**

PLL response with no loop filter

To further understand the PLL transfer function response and stability, let us examine the case when there is no loop filter. With $F(s) = 1$ (i.e. no loop filter), the PLL closed loop transfer function becomes (3),

$$\frac{\theta_0}{\theta_I} = \frac{K_P F(s) K_0 / s}{\left(1 + K_P F(s) K_0 / s\right)} = \frac{K_P K_0}{s + K_P K_0}$$

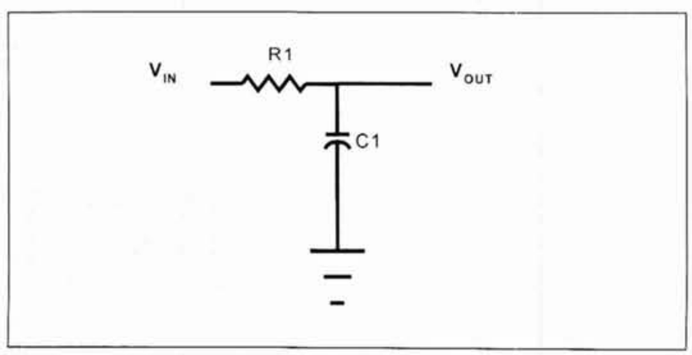


▲ **Figure 3. PLL closed loop transfer function with no loop filter.**

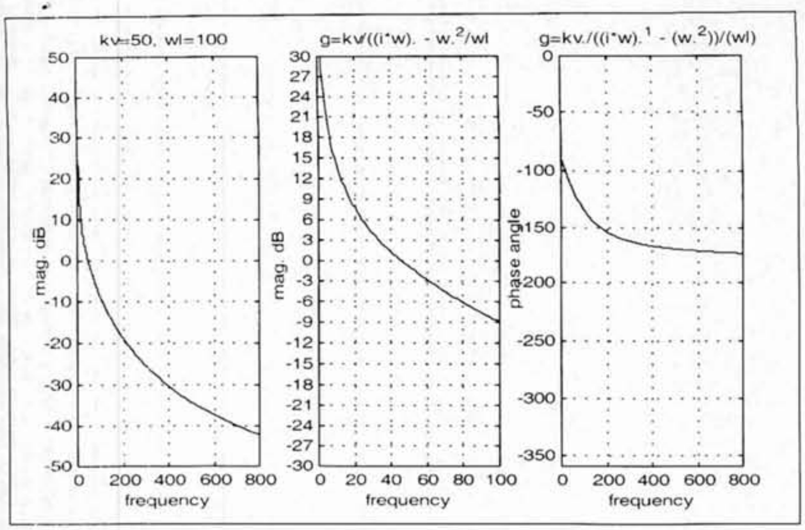
which is the transfer function of a LPF with DC gain of unity and a 3 dB cutoff frequency of $K_P \times K_0$. Therefore, an increase in the DC gain of the phase detector and/or the VCO results in a wider loop, which in turn results in higher phase noise in the PLL. Also, the open loop gain has a slope of 6 dB per octave or 20 dB per decade for all frequencies. The phase angle is always –90 degrees at all frequencies. Hence, with no low-pass filter in the loop, the PLL is always stable, according to the stability criteria. But the main drawback of a PLL design with no loop filter is that the designer has little or no control over the loop response. Figures 2 and 3 show a plot of the open and closed loop transfer function (gain and phase vs. frequency). The plot was done on MatLab using the absolute value function (called *abs*) to compute the magnitude of the transfer function and the angle function (called *angle*) to compute the phase angle. Also, the plot commands — plot (w, abs) and plot (w, angle) — were used to plot magnitude and phase vs. frequency.

PLL with a single loop filter

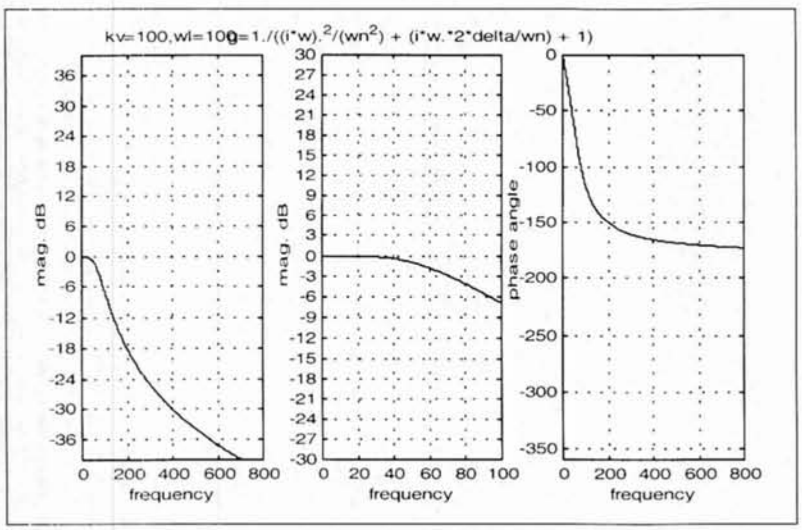
In most PLL designs, a low pass filter is used. The function of a LPF is to filter out any high-frequency harmonics in the loop that might cause the loop to go out of lock, and also to stabilize the loop. Adding a LPF also affects the loop response including parameters such as the loop time response τ_r, loop bandwidth ω_C and the damping factor ∂ of the loop. Figure 4 shows the low



▲ **Figure 4. Single pole loop filter.**



▲ **Figure 5. PLL open loop response with single pole LPF.**



▲ **Figure 6. PLL closed loop response with single pole LPF.**

pass filter that is commonly used in PLL designs. The filter is a one pole low-pass filter with a 3 dB cutoff frequency at ω_L.

Therefore, in this case,

$$F(s) = \frac{1}{s/\omega_L + 1} \qquad (4)$$

Substituting equation 4 in equation 1 gives

$$T(s) = \frac{\theta_0}{\theta_I} = \frac{K_P F(s) K_0 / s}{1 + K_P F(s) K_0 / s} = \frac{K_V / s(s/\omega_L + 1)}{1 + \dfrac{K_V}{s(s/\omega_L + 1)}} = \frac{1}{(s^2/\omega_N{}^2) + s(2\partial/\omega_N) + 1} \qquad (5)$$

where

$$K_V = K_P K_0$$
$$\omega_N = (K_V \omega_L)^{1/2}$$
$$\partial = \omega_N/(2K_V)$$

Using a low-pass filter with a cutoff frequency of ω_L, the PLL closed loop response is a 2nd order low-pass filter transfer function, centered at the VCO frequency. A characteristic of the second order low pass response is that the slope of the filter drops at a rate of 12 dB/octave. The term ω_N is defined as the natural frequency of the loop and the term ∂ is defined as the damping factor of the loop. Using equation 5 we can derive ω_C, which is

$$\omega_C = \omega_N \left(1 - 2\partial^2 + \left(2 - 4\partial^2 + 4\partial^4\right)^{1/2}\right)^{1/2}$$

for $\partial < 1$

ω_C is the 3 dB bandwidth of the PLL. Knowing ω_C, we can determine the time it takes for the PLL output to rise to 90 percent of its final value, which is approximately

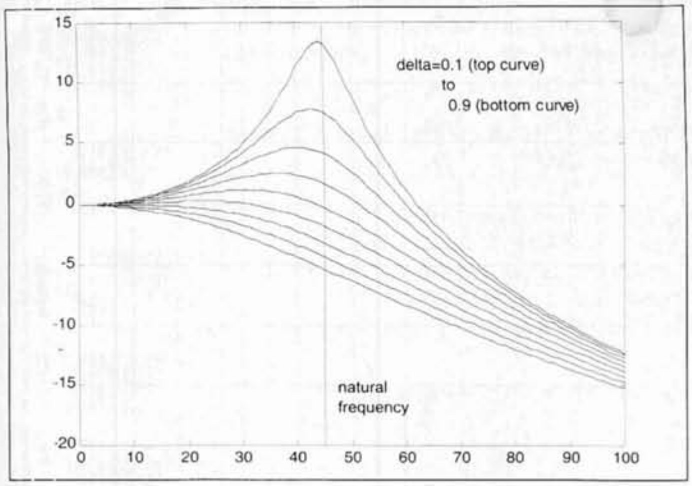
$$\tau_r = 2.2/\omega_C$$

Therefore, given the cutoff frequency ω_L of the low pass filter and the values of K_P and K_0, we can determine the natural frequency ω_N of the loop, and subsequently determine the PLL loop bandwidth ω_C. Knowing this, we can then calculate the time τ_r it takes for the PLL to settle to its final frequency value.

In order to determine the stability of the loop with a single pole low-pass filter in the loop, we must examine the open loop transfer function of the PLL. As stated before, the open loop transfer function is given by the function $G(s)$ and is equal to:

$$G(s) = K_P F(s) K_0 / s = K_P K_0 / s(s/\omega_L + 1) = \frac{K_V}{s(s/\omega_L + 1)} \qquad (6)$$

The open loop transfer function has two poles — one at DC and the other at ω_L. Note that at every pole, the gain slope drops at a slope of 6 dB per octave. The gain at DC is infinite; as the frequency increases, the magnitude of the transfer function drops at a slope of 6 dB per octave (due to the presence of the pole at DC). When the frequency reaches ω_L, the gain drops at a slope of another 6 dB per octave (a total slope of 12 dB per octave after



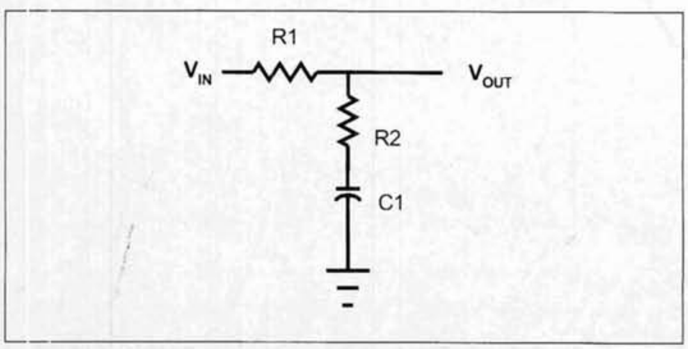
▲ **Figure 7. Closed loop gain response vs. frequency for different damping factors (¶).**

two poles). The PLL is unstable at the frequency where open loop gain crosses the unity gain line at a slope of 12 dB per octave and the phase is –180 degrees. This condition can be observed by plotting the open loop gain and phase response using MatLab. The condition of instability can be avoided by the selection of the pole frequency of the loop filter. As the pole frequency ω_L decreases, the open loop gain intersects the unity gain axis at 12 dB/ octave, the phase angle approaches –180 degrees and the PLL becomes unstable. The PLL approaches instability. This can be observed either by decreasing ω_L while observing the gain and phase response on Matlab or by examining the phase response of the open loop transfer function, which is

$$\varphi = -\tan^{-1} - (\omega_L / \omega)$$

Clearly, as ω_L decreases, the phase angle approaches –180 degrees.

One disadvantage of using a single pole filter is that both the closed loop bandwidth and the damping factor of the closed loop response of the PLL depend on the loop filter bandwidth. The designer cannot independently set the loop bandwidth without affecting the amount of transient overshoot. This deficiency can be easily overcome if a pole-zero loop filter is used. See Figures 5 and 6 for the open and closed loop response of the PLL with a single pole loop filter.



▲ **Figure 8. Pole-zero filter.**

When designing a PLL, it is important to choose the damping factor such that the loop time response has very little overshoot. The percent of overshoot is defined as the time it takes for a PLL to settle at a given frequency. A high percent overshoot can cause the loop to go out of lock. Figure 7 is a plot of the closed loop gain response vs. frequency for different values of damping (from = 0.1 to 0.9 in increments of 0.1, 0.707 being the design goal).

PLL response with a pole-zero loop filter

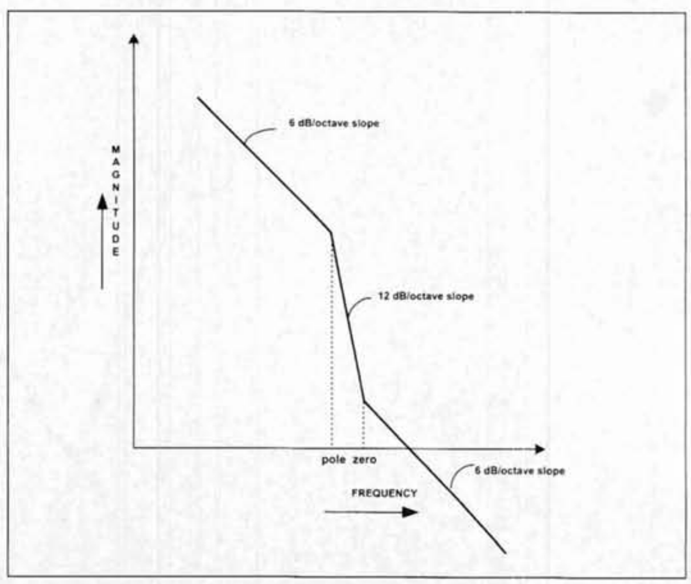
Another way to control the loop response is by using a pole-zero filter in the PLL (see Figure 8). A pole-zero filter is a low pass filter with a pole frequency ω_P and a zero frequency ω_Z. The addition of a pole in the transfer function causes the transfer function slope to drop at a rate of 6 dB per octave whereas the addition of a zero in the PLL transfer function has the opposite effect. For example, the addition of a zero frequency increases the slope by a 6 dB/ octave. This phenomenon is illustrated in Figure 9.

The pole-zero filter transfer response is given by

$$F(s) = \frac{s / \omega_z + 1}{s / \omega_p + 1} \tag{7}$$

where ω_Z is the zero frequency and ω_P is the pole frequency. The open loop transfer function is:

$$G(s) = \frac{K_V F(s)}{s} = \frac{K_V (s / \omega z + 1)}{s(s / \omega_P + 1)} \tag{8}$$



▲ **Figure 9. PLL open loop gain response with a pole-zero loop filter.**

In this case, the location of the pole is always before the zero frequency. Given the pole frequency location, a zero can be placed after the pole (as shown in the Figure 9) so as to avoid the magnitude from crossing the unity gain axis at a slope of 12 dB per octave, and therefore avoiding instability.

To determine the closed loop response, simply plot $T(s)$,

$$T(s) = \frac{\theta_0}{\theta_I} = \frac{K_P F(s) K_0 / s}{\left(1 + K_P F(s) K_0 / s\right)} \tag{9}$$

where

$$F(s) = \frac{s / \omega_Z + 1}{s / \omega_P + 1}$$

The transfer function of $T(s)$ is

$$T(s) = \frac{(s\omega_z + 1)}{s^2 / \omega_p K + (1 + K / \omega_z)s + 1} \tag{10}$$

where $\omega_N = (\omega_p K)^{1/2}$ and

$$\partial = (.5)(1/K + 1/\omega_z)\omega_N$$

Therefore, selecting the pole frequency sets the natural frequency (and subsequently the loop bandwidth) and selecting the zero (based on the pole location in the open loop gain response) determines the desired percentage overshoot. Therefore, a pole-zero filter allows the designer to select the loop bandwidth and the damping factor independently and still achieve stability.

Summary

Steady-state stability is an important criterion in PLL design. Stability can be determined by examining the transfer function of the PLL in an open state. As seen, a condition of open loop gain of unity and a phase angle of –180 degrees must be avoided for stable operation of the PLL. This can be accomplished by the proper selection of the loop filter parameters. ■

References

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