Lecture 35: Receiver Intermodulation and Dynamic Range.

On the opposite end from small signals (i.e., those near the noise floor), ***strongly*** received signals can also limit the performance of receivers.

Isn't this statement paradoxical? On the contrary: as we'll see in this lecture, two strong signals that are near to each other in frequency can create spurious receiver outputs.

These new spurs are produced when unintentional mixing occurs in a receiver amplifier or mixer. This unintentional mixing happens in the receiver "front end" (e.g., RF pre-amplifier and/or RF mixer) when:

1. Two or more signals are received that are close together in frequency, and
2. One or more of these signals is so strong that circuit components in the receiver front end no longer behave as intended.

Examples of this latter situation include large input signals that drive the semiconductor devices in a mixer into nonlinear behavior leading to unintended mixing, or a small-signal amplifier driven into nonlinear behavior so that it is no longer a linear amplifier.

As discussed next, it is possible in such a circumstance that unintended audio output signals will be produced, in addition to the intended signal. This is, by definition, a spurious output, or spur. These particular spurs are called intermodulation products.

It is important to realize that these spurs are different from those considered earlier in this course (i.e., image frequencies and mixer products).

Mathematics of Intermodulation Products

To understand the origin of intermodulation products (IPs), consider the Taylor series expansion of a "weakly nonlinear" output voltage

$$V = \underbrace{G_v V_i}_{\substack{\text{desired} \\ \text{output}}} + \underbrace{G_2 V_i^2 + G_3 V_i^3 + G_4 V_i^4 + G_5 V_i^5 + \cdots}_{\text{IPs}} \qquad (14.41)$$

The IPs occur when V_i is the sum of two (or more) **strong** signals that are close together in frequency:

$$V_i = V_1 \cos(\omega_1 t) + V_2 \cos(\omega_2 t) \qquad (14.42)$$

Let's choose $V_1 = V_2 = V$ and $\omega_1 < \omega_2$, as in the text.

We will now substitute (14.42) into (14.41) and expand each of the HO terms. We will assume that V_1 and V_2 are large enough that the HO terms in (14.41) are appreciable in size to $G_v V_i$.

- Second-order products:

$$V_i^2 = \left[V\cos(\omega_1 t) + V\cos(\omega_2 t) \right]^2 \tag{1}$$

Using *Mathematica*, we can symbolically expand and then simplify this expression:

```
Vi = V * Cos[ω1 * t] + V * Cos[ω2 * t] ;
Factor[TrigReduce[Vi]]

V (Cos[t ω1] + Cos[t ω2])

Factor[TrigReduce[Vi * Vi]]

1/2 V² (2 + Cos[2 t ω1] + Cos[2 t ω2] + 2 Cos[t ω1 - t ω2] + 2 Cos[t ω1 + t ω2])
```

From this result, we see that

$$V_i^2 = \frac{V^2}{2}\Big[2 + \cos(2\omega_1 t) + \cos(2\omega_2 t)$$
$$+2\cos(\omega_1 t - \omega_2 t) + 2\cos(\omega_1 t + \omega_2 t) \Big]$$

Now, let's imagine that we have two **strong** signals at

$$f_1 = 7.030 \text{ MHz} \quad \text{and} \quad f_2 = 7.040 \text{ MHz}$$

as in Prob. 35. IP spurs should then be located at

$2f_1 = 14.060$ MHz

$2f_2 = 14.080$ MHz,

$|f_1 - f_2| = 0.010$ MHz

$f_1 + f_2 = 14.070$ MHz

All of these IP spurs are located far from the intended RF signals and would be well attenuated by the RF Filter.

- Third-order products:

$$V_i^3 = \left[V\cos(\omega_1 t) + V\cos(\omega_2 t) \right]^3 \tag{2}$$

Again using *Mathematica*:

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Factor[TrigReduce[Vi * Vi * Vi]]

1/4 V^3 (9 Cos[t ω1] + Cos[3 t ω1] + 9 Cos[t ω2] + Cos[3 t ω2] +
    3 Cos[t ω1 - 2 t ω2] + 3 Cos[2 t ω1 - t ω2] + 3 Cos[2 t ω1 + t ω2] + 3 Cos[t ω1 + 2 t ω2])
```

Using f_1 and f_2 defined above, then the IP spurs are located at:

f_1 = 7.030 MHz (actually one fundamental)
f_2 = 7.040 MHz (the other fundamental)

$3f_1$ = 21. 090 MHz
$3f_2$ = 21.120 MHz

$2f_2 - f_1$ = 7.050 MHz (very near input *f*)
$2f_1 - f_2$ = 7.020 MHz (again, very near input *f*)

$2f_2 + f_1$ = 21.110 MHz
$2f_1 + f_2$ = 21.100 MHz

The two spurs near the intended operational frequencies of the receiver are defined in the text as

$$\boxed{f_{\underline{3}} = 2f_1 - f_2} \tag{14.37}$$

and

$$\boxed{f_{\overline{3}} = 2f_2 - f_1} \tag{14.38}$$

These two particular third-order IPs are often troublesome because they can be close in value to the frequencies that created them (when $f_1 - f_2$ is small). If f_1 and f_2 fall within the passband of the RF Filter, then it is conceivable offending IPs at $f_{\underline{3}}$ and/or $f_{\overline{3}}$ could pass through the IF Filter. In such a situation, these IPs cannot be filtered out by the receiver and spurious outputs will occur.

- Fourth-order products:

$$V_i^4 = \left[V\cos(\omega_1 t) + V\cos(\omega_2 t)\right]^4 \tag{3}$$

Again using *Mathematica*:

```
Factor[TrigReduce[Vi * Vi * Vi * Vi]]

1/8 V^4 (18 + 16 Cos[2 t ω1] + Cos[4 t ω1] + 16 Cos[2 t ω2] + Cos[4 t ω2] +
   4 Cos[t ω1 - 3 t ω2] + 6 Cos[2 t ω1 - 2 t ω2] + 24 Cos[t ω1 - t ω2] + 4 Cos[3 t ω1 - t ω2] +
   24 Cos[t ω1 + t ω2] + 4 Cos[3 t ω1 + t ω2] + 6 Cos[2 t ω1 + 2 t ω2] + 4 Cos[t ω1 + 3 t ω2])
```

Using f_1 and f_2 defined above, these IP spurs are located at:

$2f_1$ = 14.060 MHz
$2f_2$ = 14.080 MHz
$4f_1$ = 28.120 MHz
$4f_2$ = 28.160 MHz

$3f_2$ - f_1 = 14.090 MHz
$3f_1$ - f_2 = 14.050 MHz
$3f_2 + f_1$ = 28.150 MHz
$3f_1 + f_2$ = 28.130 MHz

$|2f_1 - 2f_2| = 0.020$ MHz
$2f_1 + f_2 = 21.110$ MHz

$|f_1 - f_2| = 0.010$ MHz
$f_1 + f_2 = 14.070$ MHz

None of these IPs are close to the receiver's input bandwidth, so they are easily filtered out by the RF Filter.

- Fifth-order products:

$$V_i^5 = \left[V\cos(\omega_1 t) + V\cos(\omega_2 t)\right]^5 \quad (4)$$

You will determine these IP frequencies in Prob. 35. As stated in the text, the two fifth-order IP spurs that can cause trouble in the NorCal 40A are

$$\boxed{f_{\underline{5}} = 3f_1 - 2f_2} \quad (14.39)$$

and

$$\boxed{f_{\overline{5}} = 3f_2 - 2f_1} \quad (14.40)$$

In the example here using the f_1 and f_2 specified above, then

$$f_{\underline{5}} = 7.010 \text{ MHz} \quad \text{and} \quad f_{\overline{5}} = 7.060 \text{ MHz}$$

These two spurs are within the passband of the receivertoo close to the input signal frequency to be filtered out by the RF Filter. Together with $f_{\underline{3}}$ and $f_{\overline{3}}$, this is more bad news!

- Higher-order products:

No other IP spurs are close to the input frequency, or generally do not have appreciable signal level.

Lastly, IP spurs are *always* present in a receiver. However, only when the input signals are sufficiently strong do the IPs rise above the noise floor and, perhaps, become large enough to cause audio output.

Are IP Spurs *Really* a Problem?

Intermodulation product spurs can be a real problem in a receiver. This is probably truer today than it was 30 years ago. There are two primary reasons:

1. Solid-state amplifiers are easier to drive into nonlinear behavior than tube amplifiers. From (14.41), we see that IP spurs are due to nonlinear behavior.
2. There are more RF signals today such as wireless PCS, radio stations, microwave datalinks, etc. If you are too close to a transmitter, your receiver "front end" may be driven to nonlinearity.

If you were concerned with jamming an adversary's radio or radar, perhaps you could take advantage of IP spurs. How?

Effects of Intermodulation Products

The effects of intermodulation products are illustrated below in Fig. 14.9. The slope of the “signal” and “intermodulation” plots are approximately equal to the order of the dominant IP.

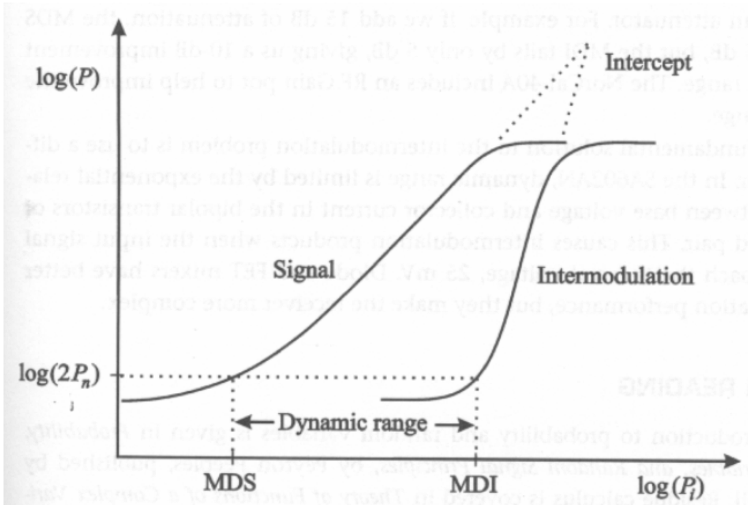


Figure 14.9. Finding the dynamic range for a receiver. Plot of the output signal and intermodulation product P versus the input power P_i on log scales. The outputs saturate at high levels because of the AGC. People often extrapolate the linear portion of the curves until they intersect. Manufacturers often quote the input or output powers associated with the intercept as a measure of the quality of the amplifier or mixer.

To understand this last statement, first note that the average signal power is expressed as

$$P = \frac{V_{rms}^2}{R} \qquad (5)$$

Next, from

$$V = \underbrace{G_v V_i}_{\text{desired output}} + \underbrace{G_2 V_i^2 + G_3 V_i^3 + G_4 V_i^4 + G_5 V_i^5 + \cdots}_{\text{IPs}} \qquad (14.41)$$

and using (1) we can deduce that for a particular IP of order n:

$$V_{n,rms} \propto V_{i,rms}^n \qquad (6)$$

This important fact can be verified from (14.43) in the text where $V \propto V^2$, from (2) above where $V \propto V^3$, etc.

Consequently, substituting (6) into (5) we find

$$P \propto \frac{\left(V_{i,rms}^{n}\right)^2}{R} = \frac{\left(V_{i,rms}^{2}\right)^n}{R} \tag{7}$$

For a log-log plot of this output power versus P_i, we need to rearrange (7) so that P_i is explicitly present:

$$\log(P) = \log\left[\frac{\left(V_{i,rms}^{2}\right)^n}{R}\frac{R^n}{R^n}\right] = \log\left[\left(\frac{V_{i,rms}^{2}}{R}\right)^n\right] + \log\left(R^{n-1}\right)$$

Simplifying gives

$$\boxed{\log(P) = n\log P_i + \log\left(R^{n-1}\right)} \tag{8}$$

This is, of course, an equation for a straight line. The slope of this P versus P_i curve equals n, which is the order of the IP. Hence, we have proven the conjecture.

Dynamic Range

The minimum detectible intermodulation input (MDI) is the input signal that produces a total output signal = $2P_n$ (signal + noise) for the *dominant* IP.

Then, by definition

$$\boxed{\text{Dynamic range} = \text{MDI} - \text{MDS} \;\; [\text{W}]} \tag{14.47}$$

and is illustrated above in Fig. 14.9.

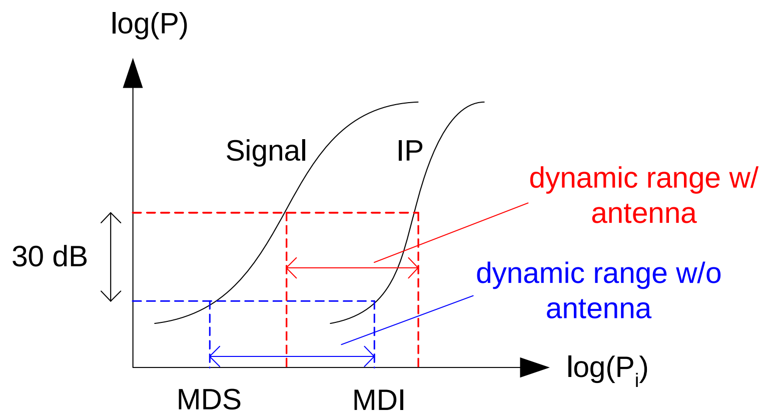
Dynamic range is:

- The range of useful input signal power levels for a receiver.
- Limited by noise for small signals and by receiver front-end nonlinearities for large signals.

Good receivers have a dynamic range ≈ 100 dB, or so.

Effects of Antenna Noise on Dynamic Range

Antenna noise can have a marked effect on dynamic range. In the NorCal 40A, the antenna noise is approximately 30 dB above the receiver noise:



From this plot, with:

- the slope of the signal curve equal to 1:1 (linear power amplification), and
- the slope of the IP curve equal to 3:1 (dominant third-order IP),

then increasing the noise floor by 30 dB due to the antenna noise causes the:

1. MDS to increase by $30\text{ dB}\cdot\frac{1}{1}=30\text{ dB}$
2. MDI to increase by $30\text{ dB}\cdot\frac{1}{3}=10\text{ dB}$

Therefore, with the antenna attached to the receiver, the dynamic range decreases by 20 dB!

Interestingly, we can retrieve some of the dynamic range by introducing attenuation at the front end, though you will sacrifice receiver sensitivity and you may decrease the loudness of the signal.

For example, if we add 15 dB of attenuation at the front end of the receiver, then

1. MDS decreases by 15 dB (= $15\text{ dB}\cdot\frac{1}{1}$),
2. MDI decreases by 5 dB (= $15\text{ dB}\cdot\frac{1}{3}$).

Consequently, the dynamic range increases by 15-5 = 10 dB. In practice, you can use your RF Gain pot to improve dynamic range if you need to.

Of course, the best way to increase dynamic range is with a better mixer design (lower noise, less susceptibility to IP). This would involve a more complicated design and likely more expense.