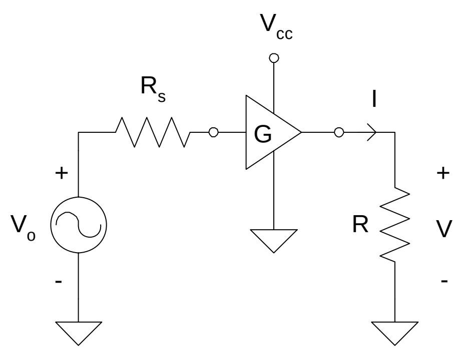
Lecture 19: Available Power. Distortion. Emitter Degeneration. Miller Effect.

While the efficiency of an amplifier, as discussed in the previous lecture, is an important quality, so is the gain of the amplifier. Transducer gain, which we simply call Gain, G, is defined as

$$G \equiv \frac{P}{P_i} \tag{1.22}$$

as we've seen previously.

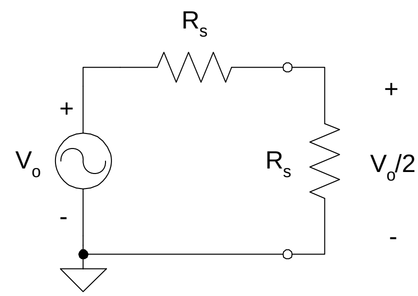
With transistor amplifiers, we want to characterize the gain of an **ac input signal** as in the following circuit:



Consequently for this amplifier, the numerator in (1.22) is the ac output power $P = V_p I_p/2$. With $V_p = V_{pp}/2$ and $I_p = I_{pp}/2$, then

$$P = \frac{V_{pp}^2}{8R} \tag{9.14}$$

Now, what about the "input" power for (1.22)? For this amplifier, we're only interested in the ac signal. The maximum ac power possible from the source V_o with a matched load as in



is the available power P_+ given by

$$\boxed{P_+ = \frac{1}{2}\frac{(V_o/2)^2}{R_s} = \frac{V_o^2}{8R_s} \quad [\text{W}]}$$

In other words, how well the amplifier and load are matched to the source dictates how much power is "available," i.e., input to the amplifier.

Recall that the displayed voltage on an AWG with a matched load is $V_{+p} = V_o/2$ (where p indicates peak). Therefore, $V_{+pp} = 2V_{+p} = V_o$ which yields

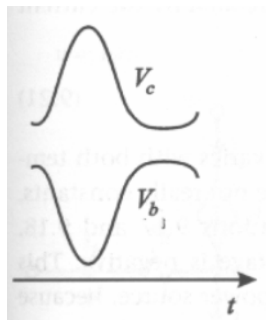
$$\boxed{P_+ = \frac{V_{+pp}^2}{8R_s} \quad [\text{W}]} \tag{9.16}$$

where V_{+pp} is the displayed peak-to-peak voltage on the AWG.

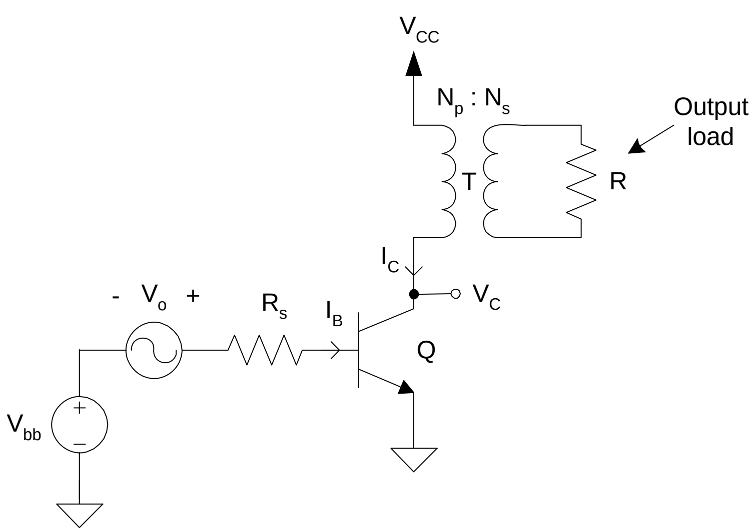
In summary, the ac gain of an amplifier in (1.22) contains the ratio of two power terms. The ac output power to a resistive load in (9.14) forms the numerator. The denominator can be defined a number of ways. Here we have chosen a conservative measure: the available power from the source, given in (9.16).

Distortion

You will most likely discover in Prob. 21 (Driver Amplifier) that when the input voltage amplitude becomes too large, the output voltage waveform will be distorted. An example is shown in Fig. 9.6a:



Recall that the Driver Amplifier is (almost) a CE amplifier with a transformer coupled resistive load:



The slight nonlinear behavior of V_c in Fig. 9.6a is due to the base-emitter diode. As illustrated in Fig. 9.7:

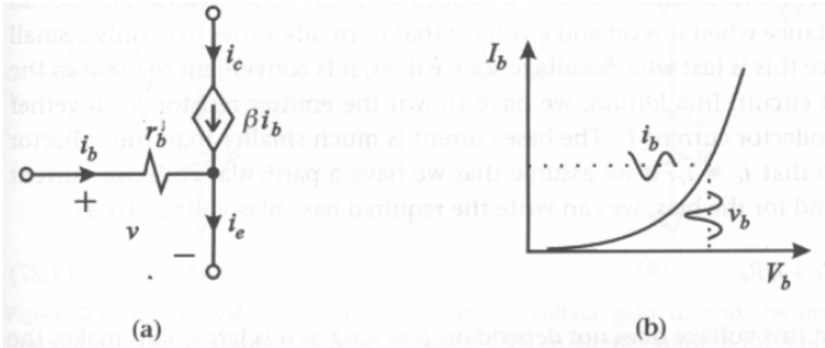


Figure 9.7. Small-signal model of an active transistor, including the base resistance r_b (a). Understanding the distortion in the base voltage waveform (b).

The distortion in Fig. 9.7b is due to the nonlinear behavior of the base-emitter junction at large signals (not because of the base resistance as stated in the text).

Other distortions you may encounter are illustrated in Fig. 9.20:

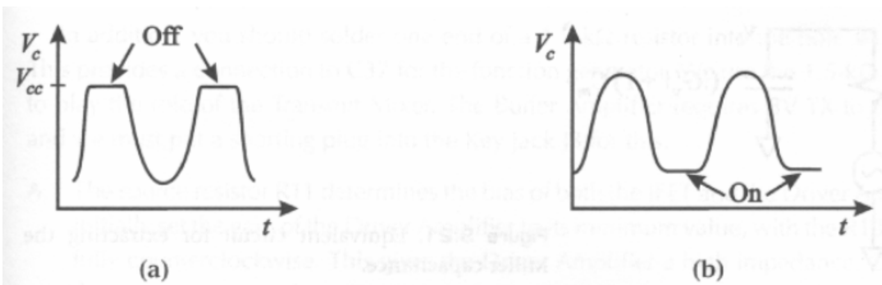


Figure 9.20. Output waveforms for setting the offset and amplitude of the function generator. In (a), the offset is too small, and in (b), the amplitude is too large.

In (a) the distortion is caused by improper input biasing, while in the (b) the distortion is from an input amplitude that is too large. (You should understand what is happening with the transistor to cause these distortions.)

Emitter Degeneration

The CE amplifiers we've considered have all had the emitter tied directly to ground. Notice that the Driver Amplifier has the additional resistance R12+R13 connected to the emitter (and eventually to ground through Key Jack J3 when transmitting).

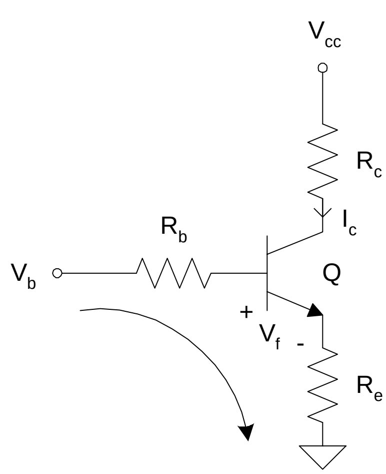
Adding an emitter resistance is called emitter degeneration. This addition has two very important and desirable effects:

1. Simpler and more reliable bias (dc),
2. Simpler and more reliable gain (ac).

Let's consider each of these points individually:

1. *Bias (dc)* – assuming an active transistor, then using KVL from V_b through R_e to ground gives

$$V_b = I_b R_b + V_f + I_e R_e$$



With $I_c \approx I_e$ then,

$$V_b \approx I_b R_b + V_f + I_c R_e$$

We will choose V_b with some I_c bias in mind ($I_c = \beta I_b$). There are two cases to consider here:

(a) $R_e = 0$: $$V_b \approx I_b R_b + V_f = I_c \frac{R_b}{\beta} + V_f .$$

Here we see that the bias current I_c will depend on the transistor β. This is not a good design since β can vary considerably among transistors.

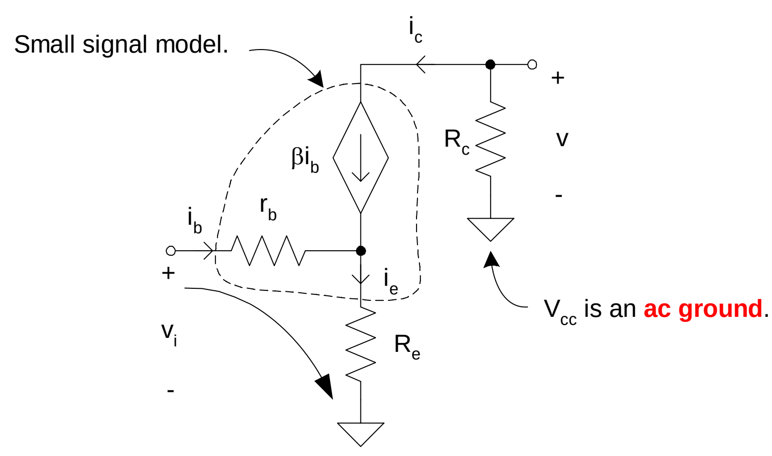
(b) $R_e \neq 0$: $$V_b \approx \cancel{I_b R_b} + V_f + I_c R_e$$

The first term is usually small wrt the third term. This leaves us with

$$V_b \approx V_f + I_c R_e$$

This is a good design since we can set V_b for a desired I_c without explicitly considering the transistor β.

2. *Gain, G* – To determine ac gain we use a small signal model of the BJT in the circuit shown above (Fig. 9.9a):



Note that we've chosen $R_b = 0$.

Using KVL in the base and emitter circuit gives

$$v_i = i_b r_b + i_e R_e$$

With $i_b r_b$ small and $i_e \approx i_c$ then

$$v_i \approx i_c R_e \tag{9.29}$$

In the collector arm,

$$v = -i_c R_c \tag{9.30}$$

Dividing (9.30) by (9.29) gives the small-signal ac gain G_v of this common-emitter amplifier to be

$$\boxed{G_v \equiv \frac{v}{v_i} = -\frac{R_c}{R_e}} \tag{9.31}$$

Notice that this gain depends only on the external resistors connected to this circuit and not on β. Hence, we can easily control G_v by changing R_c and R_e. Nice design!

Input and Output Impedance. Miller Effect.

The last topics we will consider in this lecture are the determination of the ac input and output impedances of this CE amplifier. It is important to know these values to properly match sources and loads to the amplifier.

1. *AC Input Impedance of the CE Amplifier with Emitter Degeneration.*

Referring to Fig. 9.9a again, the ac input impedance is defined as

$$Z_i \equiv \frac{v_i}{i_b} \qquad (9.32)$$

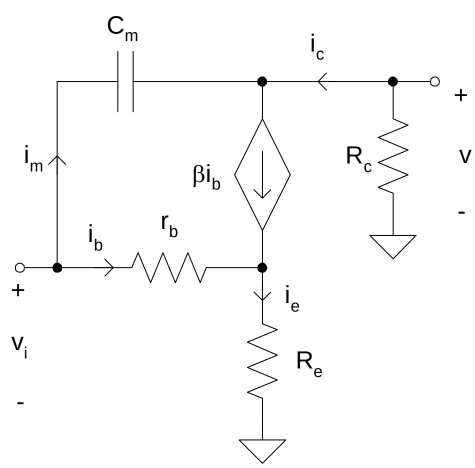
Using (9.29) and $i_c = \beta i_b$ gives

$$Z_i = \frac{i_c R_e}{i_c / \beta} = \beta R_e \quad [\Omega] \qquad (9.34)$$

Notice that Z_i is the product of two large numbers. Consequently, the ac input impedance could potentially be very large, which is desirable in certain circumstances.

However, you will see in Prob. 22 that this high input impedance is often not observed because of the so-called Miller capacitance effect.

To understand this effect, we construct the small signal model of a CE amplifier and include the base-to-collector capacitance:



This b-to-c capacitance arises due to charge separation at the CBJ. Other junction capacitances are also present in the transistor, but are not manifest at the "lower" frequencies of interest here.

While C_m, the Miller capacitance, is usually quite small (a few pF), its effect on the circuit is magnified because of its direct connection from the output to input terminals of this amplifier with high gain.

Let's now re-derive the input impedance while accounting for this Miller capacitance. Referring to the figure above, the capacitor current is

$$i_m = \frac{v_i - v}{1/(j\omega C_m)} = j\omega C_m (v_i - v) \tag{9.35}$$

From (9.31) we know that

$$\frac{v}{v_i} = -|G_v|$$

Substituting this into (9.35) we find that

$$i_m = j\omega C_m (v_i + |G_v| v_i) = j\omega C_m (|G_v| + 1) v_i \tag{9.35}$$

or

$$\frac{v_i}{i_m} = \frac{1}{j\omega \underbrace{(|G_v| + 1) C_m}_{\text{effective input capacitance}}} \tag{1}$$

We see from this expression that the effects of the capacitance C_m are magnified by the gain of the amplifier! This is the so-called Miller effect.

Therefore, considering this Miller effect the input impedance of the CE amplifier will be βR_e in parallel with the effective input capacitance from (1)

$$\boxed{Z_i = \beta R_e \,\|\, \left[j\omega\left(\left|G_v\right|+1\right)C_m \right]^{-1} \ \ [\Omega]} \tag{9.36}$$

This has the effect of **reducing** the input impedance magnitude from the huge value of βR_e.

2. *AC Output Impedance of the CE Amplifier with Emitter Degeneration.*

As shown in the text, the output impedance of a CE amplifier with emitter degeneration is given by the approximate expression

$$Z_o \approx z_c\left(1+\frac{\beta R_e}{R_s' + R_e}\right) \ \ [\Omega] \tag{9.46}$$

where

$$R_s' = R_s + r_b \ , \tag{9.38}$$

R_s is the source resistance and z_c is the collector impedance

$$z_c = r_c \,\|\, Z_c = r_c \,\|\, \left(j\omega C_c \right)^{-1} \tag{9.39}$$

This collector impedance is the parallel combination of the finite output resistance r_c of the BJT (from the Early effect illustrated in Fig. 9.10) and the finite output capacitance of the BJT, labeled C_c in the text.

The output impedance Z_o in (9.46) is often very large for CE amplifiers with emitter degeneration, which makes for a good current source.