Lecture 11: Ladder Filters. Butterworth and Chebyshev Filters. Filter Tables. *ADS*.

Ladder filters are networks that are composed of alternating series and shunt elements.

Figure 1:

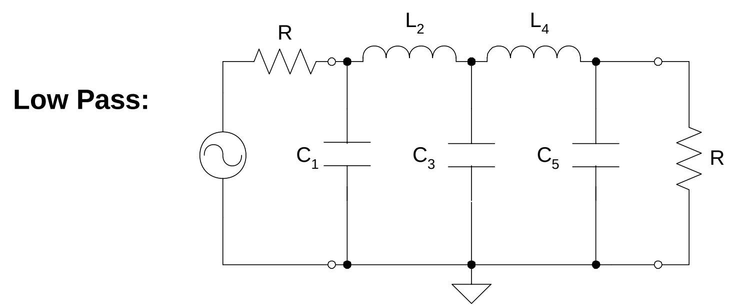
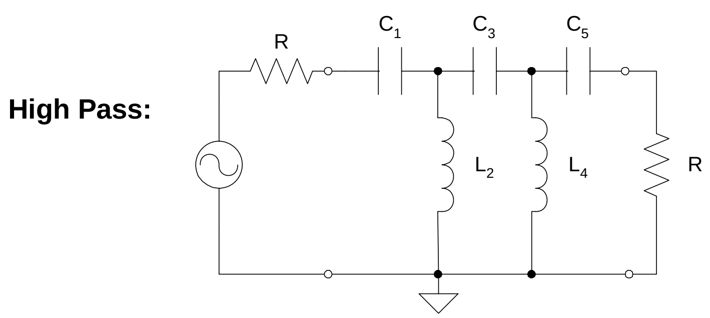


Figure 2 :



Notice that the same source and load resistances are assumed. This is called "doubly terminated" filters. All of our filters will be doubly terminated.

Ladder filters are actually one of the oldest types of filters. They have been around since the mid-1800's.

A circuit designer can achieve a sharper (or steeper) frequency roll off with ladder filters than with simple RC or RL circuits. Consequently, one can obtain more ideal low, high or band pass filter responses and with little resistive loss.

Additionally, doubly terminated ladder filters have a low sensitivity to component variation. That is a good characteristic.

There are four basic types of ladder filters:

1. Maximally flat, also called Butterworth filters,
2. Equal ripple, also called Chebyshev filters,
3. Elliptic, also called Cauer filters,
4. Linear phase filters.

We will consider the first two in this course.

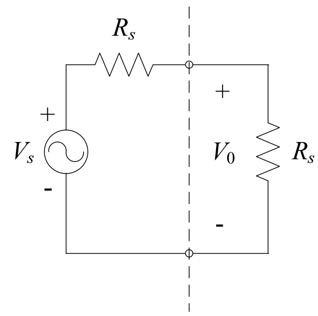
The circuits in Figs. 1 and 2 can be either Butterworth or Chebyshev filters. The topology is the same for both. Only the values for L and C vary between the two types of filters.

We will characterize these two filter types by the response of the loss factor $L(f)$ magnitude versus frequency. [The loss factor is sometimes referred to as the insertion loss = IL = 10 $\log_{10}(L)$.]

Maximum Available Power

Before further discussion of ladder filters, we must first define maximum available power, P_+. This is the maximum time average power that can be provided by a source, or by the previous stage in the circuit, to a *matched* load.

Consider that a source or previous circuit stage has been modeled by this Thévenin equivalent circuit:



As you determined in homework prob. 1, a dc source delivers maximum power when a resistive load R_s is connected to the output, similar to that shown above. For the ac circuit shown here, the maximum power delivered to the load R_s is

$$P_+ = \frac{1}{2}\frac{V_0^2}{R_s} = \frac{1}{2}\frac{\left(\frac{V_s}{2}\right)^2}{R_s} \quad \text{or} \quad \boxed{P_+ = \frac{V_s^2}{8R_s} \quad [\text{W}]} \tag{1}$$

In summary, P_+ is the maximum available power from an ac source (or a Thévenin equivalent) with internal resistance R_s. It is the maximum time-average power that can be delivered to a <u>matched source</u>. Very important formula. (Note that V_s is the amplitude, not p-to-p.)

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1. Maximally Flat, or Butterworth, Low Pass Filter

For this filter, the values of the inductors and capacitors are somehow chosen so that

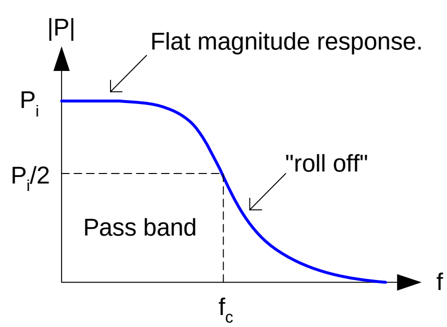
$$L_B(f) = \frac{P_i}{P(f)} = 1 + \left(\frac{f}{f_c}\right)^{2n} \qquad (5.1)$$

Where L_B is the loss factor as a function of f.

In this expression:

- P_i = maximum available power from the source (see Lecture 10),
- P = delivered power to the load,
- f_c = cutoff frequency of the filter,
- n = order of the filter (number of L's ***and*** C's in high and low pass filter; number of L-C pairs in bandpass filters).

For the Butterworth (maximally flat) low pass filter (Fig. 5.2a):



2. Equal Ripple, or Chebyshev Low Pass Filter

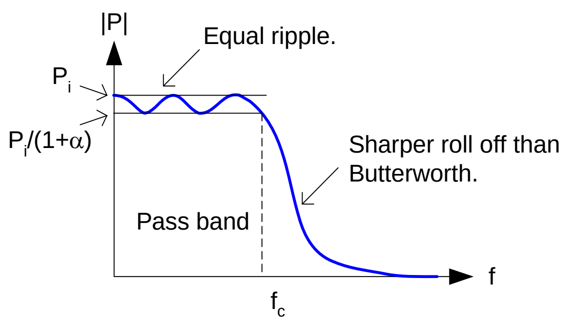
The values of the inductors and capacitors in this type of filter are <u>somehow</u> chosen so that

$$L_C(f) = \frac{P_i}{P} = 1 + \alpha C_n^2 \underbrace{\left(f/f_c \right)}_{\text{argument}} \quad (5.3)$$

In this expression:

- α = ripple size,
- $C_n\left(f/f_n \right)$ Chebyshev polynomial of order n (see plots in Fig. 5.3b),

Chebyshev filters might be more susceptible to variations in component values than Butterworth filters. This is due to the large coefficients of the polynomials listed in Fig. 5.3.



Comments

- Whether to use Butterworth, Chebyshev or another filter type depends on the specifications/requirements of the circuit (required rejection, roll off, phase variation, etc.), the available components, component value variations and so on.
- Once you have the specifications, then you can synthesize the filter. The required filter specifications are:

<table>
<tr><td rowspan="4">Chebyshev</td><td rowspan="3">Butterworth</td><td>Cutoff frequency, f_c</td></tr>
<tr><td>Order of the filter, n (for rejection)</td></tr>
<tr><td>Impedance level, R (for source and load)</td></tr>
<tr><td></td><td>Ripple in passband</td></tr>
</table>

- With these specifications, you can calculate the specific inductor and capacitor values needed to realize the filter (i.e., "synthesize" it). It is a complicated procedure to derive the formulas for these component values. There are entire books devoted to this topic. (See the attachment at the end of this lecture for a simple example.)

- Instead of deriving these formulas, designers often simply use filter tables. These are tabulated values for normalized susceptance and reactance (collectively called immittance, *a*).

  To un-normalize values from filter tables for low pass filters, use

$$L = \left(\frac{R}{R_N}\right)\left(\frac{\omega_N}{\omega_c}\right) a \quad \text{[H]}$$

$$C = \left(\frac{R_N}{R}\right)\left(\frac{\omega_N}{\omega_c}\right) a \quad \text{[F]}$$

R_N and ω_N are the normalization values used in the tables (often both = 1), while R and ω_c are the actual circuit values. An example will help explain this procedure.

Example

Design a fifth-order, Butterworth, low-pass filter (see Fig. 1 above) with a cutoff frequency of 8 MHz, a rejection of at least 23 dB at 14 MHz and an impedance level of 50 Ω.

With a fifth order filter, $n = 5$. From (5.1) and f/f_c = 14/8 then

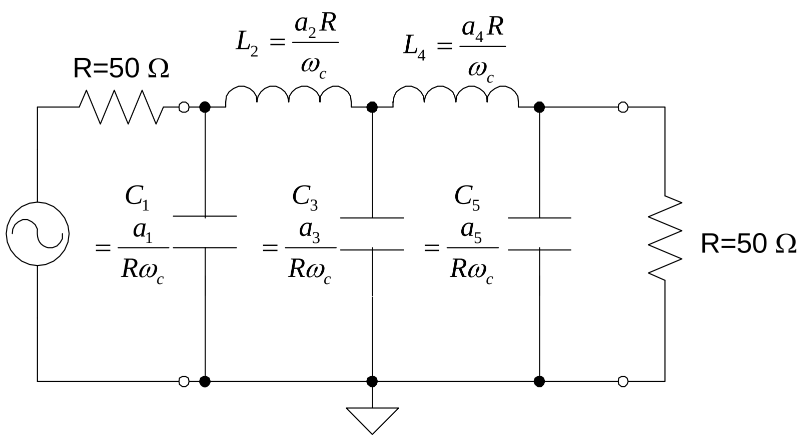
$$\text{IL}\Big|_{14\text{ MHz}} = 10\log_{10}\left(\text{L}\right) = 10\log_{10}\left[1+\left(\frac{14}{8}\right)^{2\cdot 5}\right] = 24.3\text{ dB}$$

which meets the 23 dB spec. (Note that there is also loss in the passband. At 7 MHz, for example, IL = 10 $\log_{10}[1+(7/8)^{10}]$=1.0 dB. Where does this "lost" energy go?)

Now, for this fifth-order Butterworth filter we read the immittance coefficients from Table 5.1 to be

$$a_1 = 0.618,\ a_2 = 1.618,\ a_3 = 2,\ a_4 = 1.618 \text{ and } a_5 = 0.618.$$

For a low pass filter, these immittance coefficients are the normalized susceptances of the shunt elements at f_c and the normalized reactances of the series elements at f_c.



For R = 50 Ω and $\omega_c = 2\pi f_c = 5.027 \times 10^7$ rad/s (at 8 MHz), then

- $C_1 = \dfrac{a_1}{R\omega_c} = 2.46 \times 10^{-10}$ F = 246 pF
- $L_2 = \dfrac{a_2 R}{\omega_c} = 1.61 \times 10^{-6}$ H = 1.61 μH
- $C_3 = \dfrac{a_3}{R\omega_c} = 7.96 \times 10^{-10}$ F = 796 pF (use standard 820 pF)
- $L_4 = L_2$ = 1.61 μH
- $C_5 = C_1$ = 246 pF.

All of these values are "in the ballpark" for the Harmonic Filter.

Of course, one generally needs to use standard values of components for the filter, unless you build your own inductors and/or capacitors. Consequently, the circuit may need to be "tweaked" after completing this synthesis step.

Advanced Design System **(*ADS*)**

This tweaking process can be performed using analysis software such as SPICE, *Puff* or *Advanced Design System* (*ADS*).

Your text uses the passive microwave circuit simulator called *Puff*, which comes with your text. It is DOS-based and requires the use of "scattering parameters" to characterize the behavior of circuits, including filters. (*S* parameters are discussed extensively in EE 481 *Microwave Engineering*.)

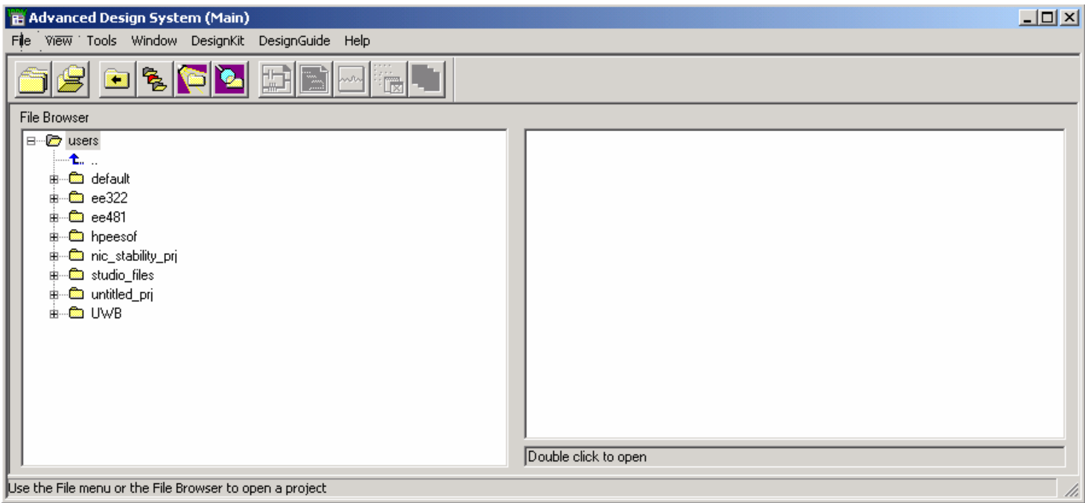
For these, and other, reasons we will **NOT** be using *Puff* in this course. Instead, we will be using *Advanced Design System* (*ADS*) from Agilent Technologies. Consequently, all of the text problems that refer to *Puff* have been rewritten to use *ADS*. These can be found on the course web site.

The manual "Getting Started with *ADS*" has been written to help you get going with *ADS*. It can also be found on the course web site. *ADS* has just a couple of nuances. Other than that, it is very straightforward to use.

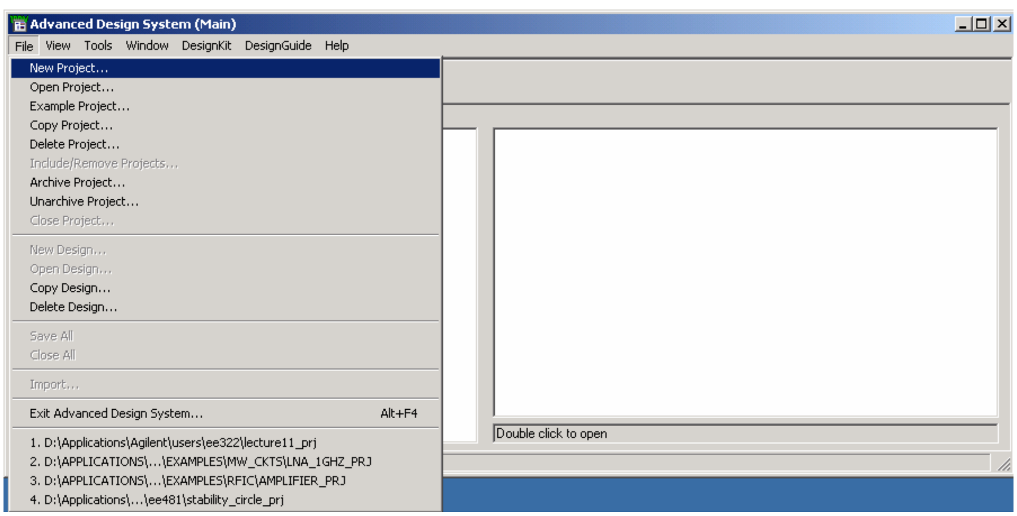
To illustrate the use of *ADS*, we will verify the proper operation of the low-pass filter designed previously.

ADS Simulation of a Low-Pass Ladder Filter

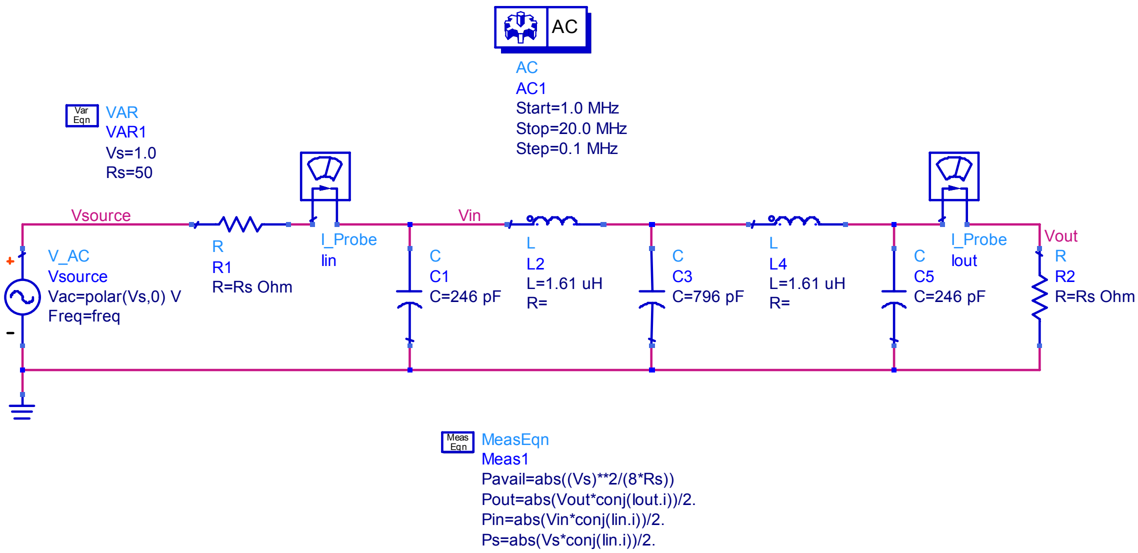
ADS Startup Window:



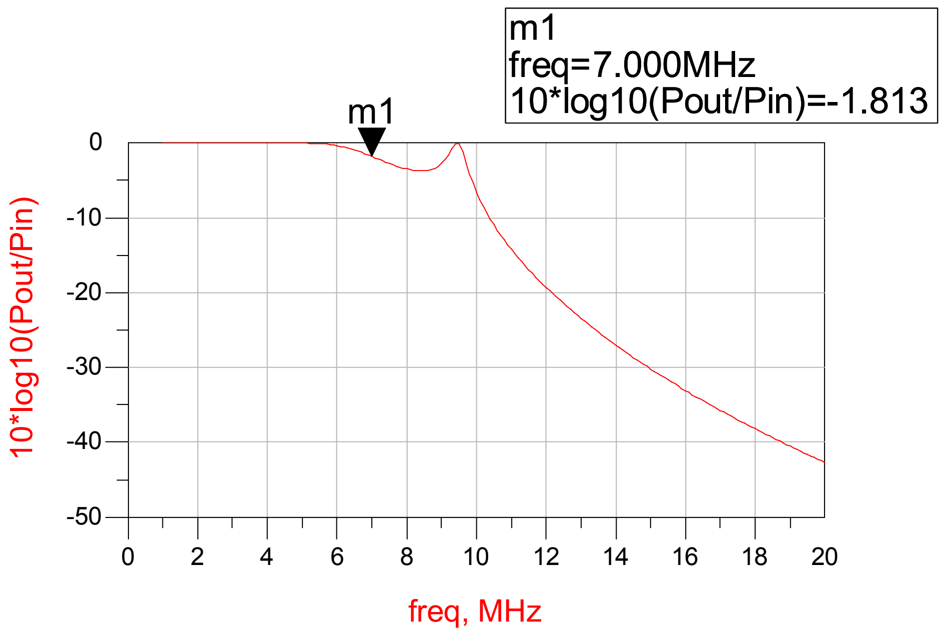
To get going with *ADS*, you must first create a "project":



ADS example with R_s = 50 Ω:

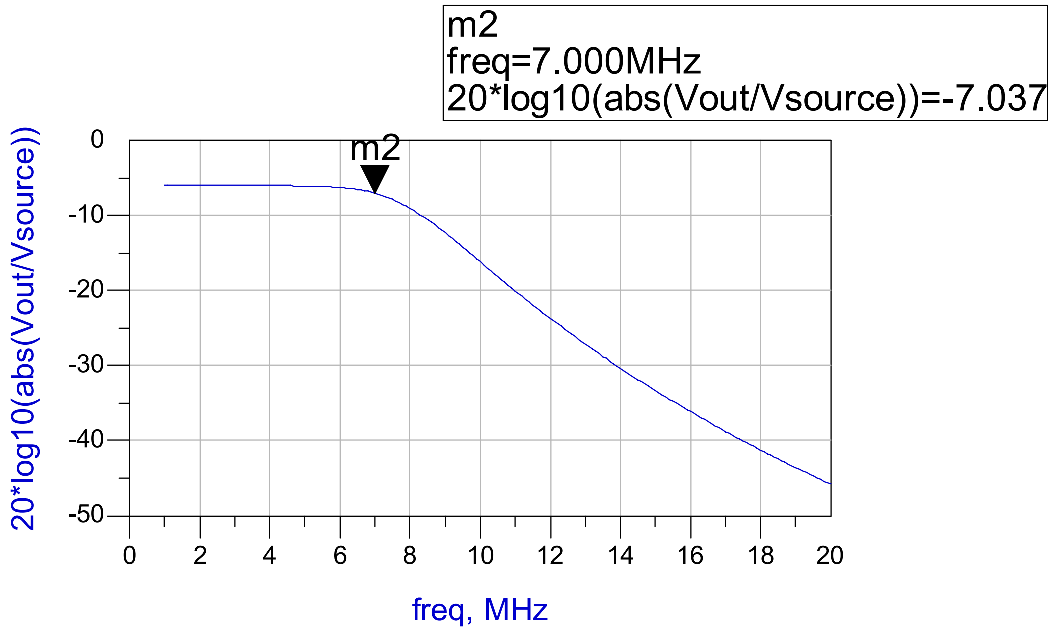


Here is a plot of P_{out}/P_{in} in dB:



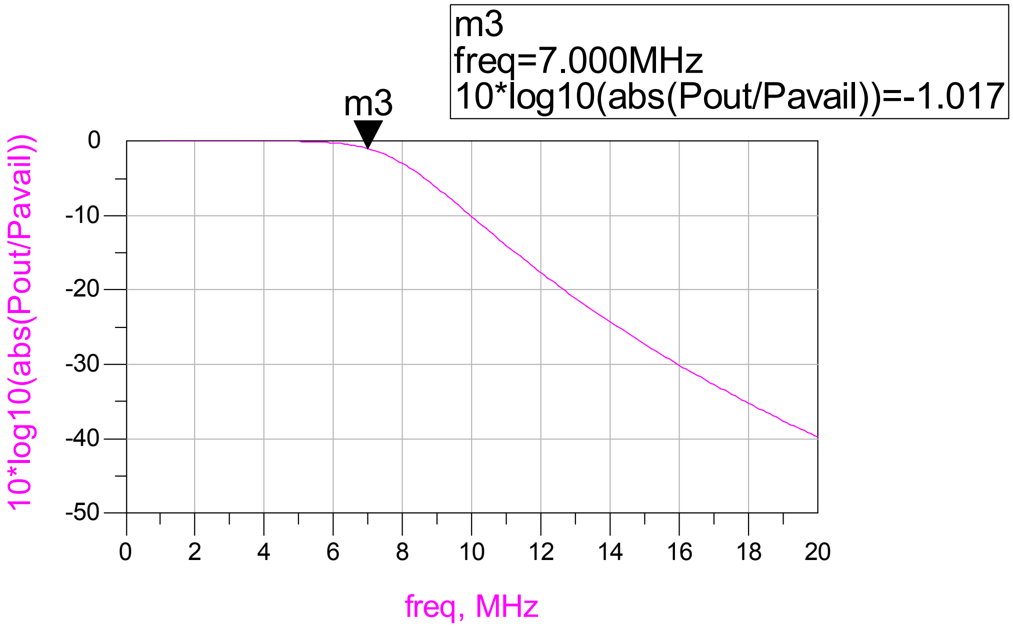
This doesn't look like the response of a maximally flat low pass filter. What's wrong?

Here's a plot of $\left|V_{\text{out}}/V_{\text{in}}\right|$ in dB:



This plot has the general shape of a maximally flat filter, but there is an extra 6 dB of attenuation at the design frequency of 7 MHz. What's going on here?

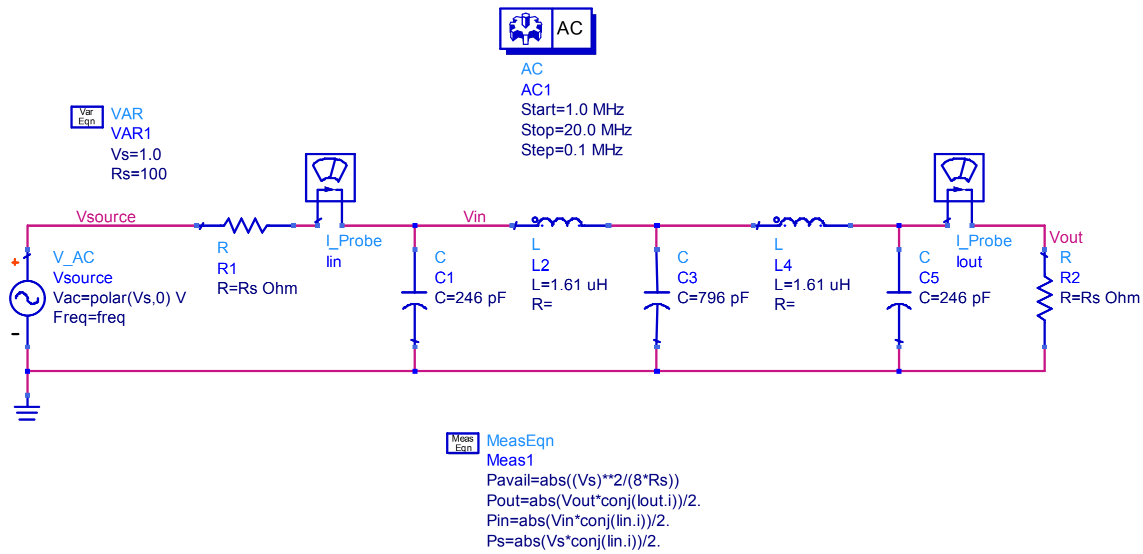
Lastly, here's a plot of P_{out}/P_{+} in dB where P_{+} is the maximum available power from the source:

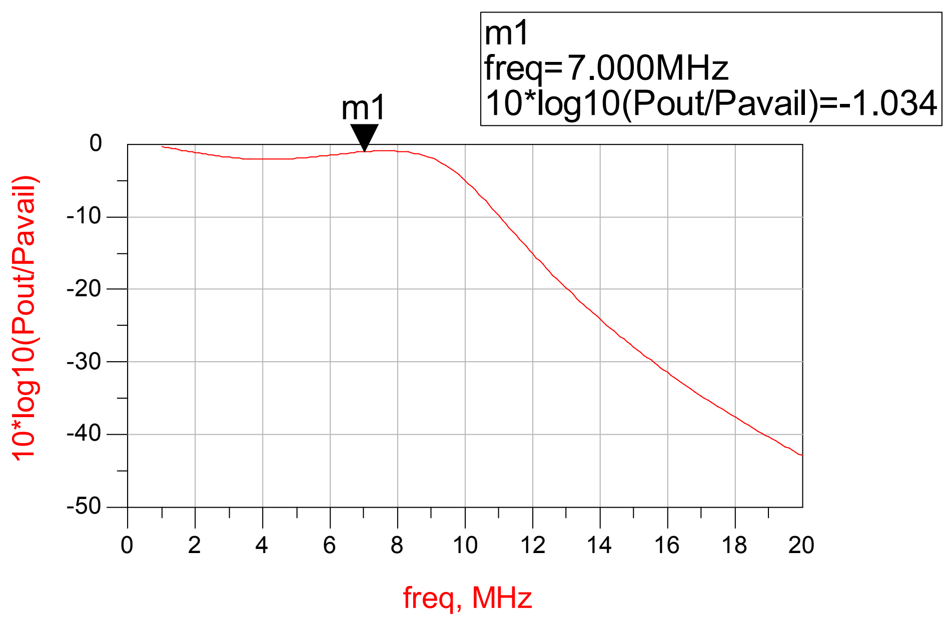


Alas, this is the plot we've been looking for. Why? Because by definition, insertion loss is the ratio of the output power to the maximum avaliable source power. See (5.1) as an example.

From this last plot, we can see that *ADS* predicts an insertion loss of –1.017 dB at 7.000 MHz. This is very close to our design prediction of –1.0 dB at 7 MHz.

ADS example with R_s = 100 Ω:





Changing the impedance "level" (from 50 Ω to 100 Ω) has a dramatic effect on the performance of the filter. Can you explain why?

From David M. Pozar, *Microwave Engineering*. New York: John Wiley & Sons, second ed., 1998:

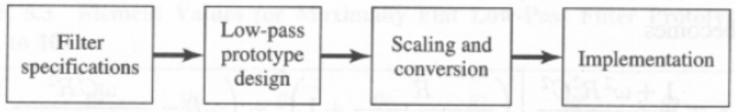


FIGURE 8.23 The process of filter design by the insertion loss method.

Maximally Flat Low-Pass Filter Prototype

Consider the two-element low-pass filter prototype shown in Figure 8.24; we will derive the normalized element values, L and C, for a maximally flat response. We assume a source impedance of 1 Ω, and a cutoff frequency $\omega_c = 1$. From (8.53), the desired power loss ratio will be, for $N = 2$,

$$P_{\mathrm{LR}} = 1 + \omega^4. \tag{8.57}$$

The input impedance of this filter is

$$Z_{\mathrm{in}} = j\omega L + \frac{R(1 - j\omega RC)}{1 + \omega^2 R^2 C^2}. \tag{8.58}$$

Since $\Gamma = \dfrac{Z_{\mathrm{in}} - 1}{Z_{\mathrm{in}} + 1}$,

the power loss ratio can be written as

$$P_{\mathrm{LR}} = \frac{1}{1 - |\Gamma|^2} = \frac{1}{1 - [(Z_{\mathrm{in}} - 1)/(Z_{\mathrm{in}} + 1)][(Z_{\mathrm{in}}^* - 1)/(Z_{\mathrm{in}}^* + 1)]} = \frac{|Z_{\mathrm{in}} + 1|^2}{2(Z_{\mathrm{in}} + Z_{\mathrm{in}}^*)}. \tag{8.59}$$

Now, $\quad Z_{\mathrm{in}} + Z_{\mathrm{in}}^* = \dfrac{2R}{1 + \omega^2 R^2 C^2},$

and $\quad |Z_{\mathrm{in}} + 1|^2 = \left(\dfrac{R}{1 + \omega^2 R^2 C^2} + 1\right)^2 + \left(\omega L - \dfrac{\omega C R^2}{1 + \omega^2 R^2 C^2}\right)^2,$

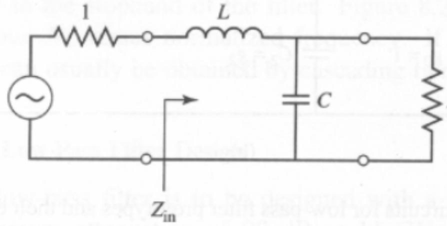


FIGURE 8.24 Low-pass filter prototype, $N = 2$.

so (8.59) becomes

$$P_{LR} = \frac{1+\omega^2R^2C^2}{4R}\left[\left(\frac{R}{1+\omega^2R^2C^2}+1\right)^2+\left(\omega L-\frac{\omega CR^2}{1+\omega^2R^2C^2}\right)^2\right]$$

$$= \frac{1}{4R}(R^2+2R+1+R^2\omega^2C^2+\omega^2L^2+\omega^4L^2C^2R^2-2\omega^2LCR^2)$$

$$= 1+\frac{1}{4R}\left[(1-R)^2+(R^2C^2+L^2-2LCR^2)\omega^2+L^2C^2R^2\omega^4\right]. \quad 8.60$$

Notice that this expression is a polynomial in ω^2. Comparing to the desired response of (8.57) shows that $R = 1$, since $P_{LR} = 1$ for $\omega = 0$. In addition, the coefficient of ω^2 must vanish, so

$$C^2 + L^2 - 2LC = (C-L)^2 = 0,$$

or $L = C$. Then for the coefficient of ω^4 to be unity we must have

$$\frac{1}{4}L^2C^2 = \frac{1}{4}L^4 = 1,$$

or

$$L = C = \sqrt{2}.$$

In principle, this procedure can be extended to find the element values for filters with an arbitrary number of elements, N, but clearly this is not practical for large N. For a normalized low-pass design where the source impedance is 1 Ω and the cutoff frequency is $\omega_c = 1$, however, the element values for the ladder-type circuits of Figure 8.25 can

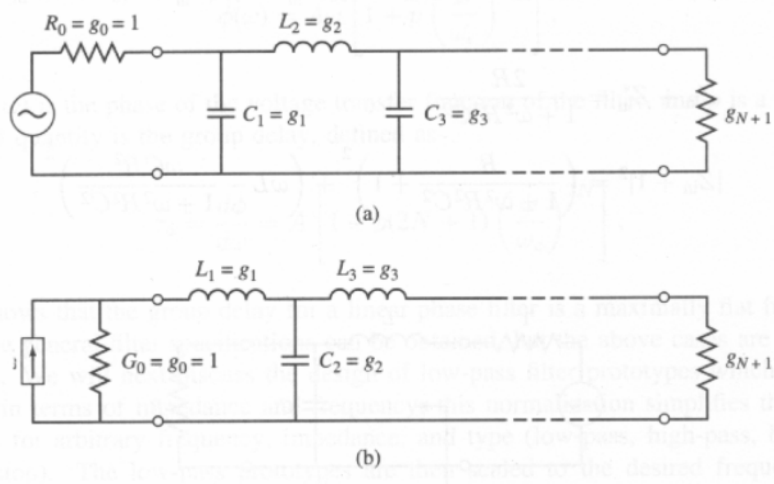


FIGURE 8.25 Ladder circuits for low-pass filter prototypes and their element definitions. (a) Prototype beginning with a shunt element. (b) Prototype beginning with a series element.

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures* (Dedham, Mass.: Artech House, 1980) with permission.

be tabulated [1]. Table 8.3 gives such element values for maximally flat low-pass filter prototypes for $N = 1$ to 10. (Notice that the values for $N = 2$ agree with the above analytical solution.) This data is used with either of the ladder circuits of Figure 8.25 in the following way. The element values are numbered from g_0 at the generator impedance to g_{N+1} at the load impedance, for a filter having N reactive elements. The elements alternate between series and shunt connections, and g_k has the following definition:

$$g_0 = \begin{cases} \text{generator resistance (network of Figure 8.25a)} \\ \text{generator conductance (network of Figure 8.25b)} \end{cases}$$

$$\underset{(k=1 \text{ to } N)}{g_k} = \begin{cases} \text{inductance for series inductors} \\ \text{capacitance for shunt capacitors} \end{cases}$$

$$g_{N+1} = \begin{cases} \text{load resistance if } g_N \text{ is a shunt capacitor} \\ \text{load conductance if } g_N \text{ is a series inductor} \end{cases}$$

Then the circuits of Figure 8.25 can be considered as the dual of each other, and both will give the same response.

Finally, as a matter of practical design procedure, it will be necessary to determine the size, or order, of the filter. This is usually dictated by a specification on the insertion loss at some frequency in the stopband of the filter. Figure 8.26 shows the attenuation characteristics for various N, versus normalized frequency. If a filter with $N > 10$ is required, a good result can usually be obtained by cascading two designs of lower order.

EXAMPLE 8.3 Low-Pass Filter Design

A maximally flat low-pass filter is to be designed with a cutoff frequency of 8 GHz and a minimum attenuation of 20 dB at 11 GHz. How many filter elements are required?